# Searching for New Physics in the $b \to d$ FCNC using $B_s^0(t) \to \phi K_S$ and $B_d^0(t) \to K^0\overline{K^0}$

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## Abstract

Time-dependent measurements of the decays $B_s^0(t) \to \phi K_s$ and $B_d^0(t) \to K^0\overline{K^0}$ can be used to compare the weak phases of $B_d^0$-$\overline{B_d^0}$ mixing and the $t$-quark contribution to the $b \to d$ penguin. Since these two phases are equal in the standard model, any discrepancy would be a sign of new physics, specifically in the $b \to d$ flavour-changing neutral current (FCNC). The method can be applied to other pairs of decays, such as $B_s^0(t) \to J/\Psi K_s$ and $B_d^0(t) \to J/\Psi\pi^0$.

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# 1 Introduction

Over the past decade or so, many methods have been proposed for obtaining the three interior angles of the unitarity triangle, $\alpha$, $\beta$ and $\gamma$. In the near future these CP phases will be measured in a variety of experiments at $B$-factories, HERA-B, and hadron colliders. As always, the hope is that these measurements will reveal the presence of physics beyond the standard model (SM).

The CP angles are typically extracted from CP-violating rate asymmetries in $B$ decays [1]. New physics, if present, will affect these asymmetries principally through new contributions to loop-level processes. Most asymmetries involve the tree-level decays of neutral mesons ($B_d^0$ or $B_s^0$), and the new physics can enter into $B^0$-$\overline{B^0}$ mixing [2]. However, it is occasionally the case that penguin contributions are involved, and the new physics can enter here as well [3].

The canonical decay modes for measuring $\alpha$ and $\beta$ are $B_d^0(t) \rightarrow \pi^+\pi^-$ and $B_d^0(t) \rightarrow J/\Psi K_s$, respectively. There have been many proposals for measuring the angle $\gamma$. One of these, accessible at asymmetric $e^+e^-$ $B$-factories, involves the CP asymmetry in $B^\pm \rightarrow DK^\pm$ [4]. Another method, which is more appropriate to hadron colliders, uses $B_s^0(t) \rightarrow D_s^\pm K^\mp$ [5].

How will we know whether or not new physics is present? One obvious way is if the three angles do not add up to 180°. Unfortunately, if the CP-angles are obtained in the conventional ways described above, $B$-factories can never find $\alpha + \beta + \gamma \neq \pi$. The reason is as follows: if there is new physics in $B_d^0$-$\overline{B_d^0}$ mixing, the CP asymmetries in $B_d^0(t) \rightarrow \pi^+\pi^-$ and $B_d^0(t) \rightarrow J/\Psi K_s$ will both be affected, but in opposite ways: instead of measuring $\alpha$ and $\beta$, the true (SM) CP-angles, one will extract $\tilde{\alpha} = \alpha - \theta_{NP}$ and $\tilde{\beta} = \beta + \theta_{NP}$, where $\theta_{NP}$ is the new-physics phase [6]. On the other hand, since the measurement of $\gamma$ does not involve $B_d^0$ decays, it will be unaffected by new physics. Thus, even in the presence of new physics, one will still find $\tilde{\alpha} + \tilde{\beta} + \gamma = \pi$.

$B$-factories must therefore find other ways of testing for the presence of new physics [7]. The most common method is simply to compare the unitarity triangle constructed from measurements of the angles with that constructed from independent measurements of the sides. If there is a discrepancy, one can then deduce that new physics is present, probably in $B_d^0$-$\overline{B_d^0}$ mixing. The problem here is that there are large theoretical errors in obtaining the length of the sides of the unitarity triangle from the experimental data. Because of this, the presently-allowed region for the unitarity triangle is still rather large [8]. Thus, if $\theta_{NP}$ is relatively small, one may still find agreement in the unitarity triangles constructed from the angles and

sides. Furthermore, even if there is a discrepancy, one isn't sure whether new physics really is present — it may simply be that the errors on the theoretical input quantities have been underestimated. The point here is that we would really like to find a method of directly probing new physics in $B_d^0$-$\overline{B_d^0}$ mixing.

Note that there are a variety of ways of testing for new physics in $B_s^0$-$\overline{B_s^0}$ mixing, or, more precisely, in the $b \to s$ flavour-changing neutral current (FCNC). For example, above we mentioned two methods for obtaining $\gamma$: via $B^{\pm} \to DK^{\pm}$ and $B_s^0(t) \to D_s^{\pm}K^{\mp}$. If there is a discrepancy in the value of $\gamma$ obtained from these two decays, this would be a direct indication of new physics in $B_s^0$-$\overline{B_s^0}$ mixing. A second example, somewhat different, is to measure $\beta$ using the decay $B_d^0(t) \to \phi K_s$ [9]. This is a pure $b \to s$ penguin decay. Thus, if the values of $\beta$ extracted via $B_d^0(t) \to J/\Psi K_s$ and $B_d^0(t) \to \phi K_s$ were to disagree, this would indicate the presence of new physics in the $b \to s$ penguin, i.e. in the $b \to s$ FCNC. (There might also be new physics in $B_d^0$-$\overline{B_d^0}$ mixing, but the effect would be the same for the two decays.) Since new physics which affects $B_s^0$-$\overline{B_s^0}$ mixing is also likely to affect the $b \to s$ penguin, in some sense this is a way of probing new physics in $B_s^0$-$\overline{B_s^0}$ mixing without actually using $B_s^0$ mesons. Finally, a third example involves the CP asymmetry in $B_s^0(t) \to J/\Psi\phi$. To a good approximation, this asymmetry vanishes in the SM, so that a nonzero value would be clear evidence of new physics, specifically in $B_s^0$-$\overline{B_s^0}$ mixing.

Although there are many ways of getting at new physics in the $b \to s$ FCNC, to date no method has been suggested which directly tests for new physics in the $b \to d$ FCNC[4]. In this Letter, we propose a method for doing just this. Essentially, the technique compares the weak phase of $B_d^0$-$\overline{B_d^0}$ mixing with that of the $t$-quark contribution to the $b \to d$ penguin. In the SM, these phases are the same, since they involve the same Cabibbo-Kobayashi-Maskawa (CKM) [12] matrix elements $V_{tb}^*V_{td}$. However, if there is new physics in the $b \to d$ FCNC, there may be a discrepancy. The method involves the decay $B_s^0 \to \phi K_s$, along with its quark-level flavour-$SU(3)$ counterpart, $B_d^0 \to K^0\overline{K^0}$. Our test of new physics in the $b \to d$ FCNC is not entirely clean – it involves some theoretical input. However, the assumption we make is reasonably well-motivated, and so this may provide a first direct probe for new physics in the $b \to d$ FCNC. We also discuss a variation of this method involving the decays $B_s^0 \to J/\Psi K_s$ and $B_d^0 \to J/\Psi\pi^0$.

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[4]In Ref. [10] it was claimed that the study of the Dalitz plot of $B_d^0(t) \to \pi^+\pi^-\pi^0$ decays allows one to cleanly perform such tests. However, it has since been shown that this particular point is in error, see Ref. [11].

# 2  $B_s^0(t) \to \phi K_S$ and $B_d^0(t) \to K^0 \overline{K^0}$

When discussing the weak phases probed in various CP asymmetries, it is convenient to use approximate Wolfenstein parametrization of the CKM matrix [13], in which only $V_{td}$ and $V_{ub}$ have significant non-zero phases. The CKM phases in the unitarity triangle are then $\beta = \mathrm{Arg}(V_{td}^*)$ and $\gamma = \mathrm{Arg}(V_{ub}^*)$, with $\alpha$ defined to be $\pi - \beta - \gamma$. We will use this parametrization throughout the paper.

We begin by considering the decay $B_s^0 \to \phi K_S$. The CP asymmetry in $B_s^0(t) \to \phi K_S$ measures the relative phases of the two amplitudes $(B_s^0 \to \phi K_S)$ and $(B_s^0 \to \overline{B_s^0})(\overline{B_s^0} \to \phi K_S)$. To begin with, let us assume that there is no new physics. $B_s^0 \to \phi K_S$ is a pure penguin decay, which at the quark level takes the form $\bar{b} \to \bar{d} s \bar{s}$. Suppose first that this decay is dominated by an internal $t$-quark, in which case the CKM matrix-element combination involved is $V_{tb}^* V_{td}$. Since $B_s^0$-$\overline{B_s^0}$ mixing involves $(V_{tb}^* V_{ts})^2$, the CP asymmetry probes

$$\mathrm{Arg}\left[\frac{V_{tb}^* V_{td}}{(V_{tb}^* V_{ts})^2 V_{tb} V_{td}^*}\right] = -2\beta . \tag{1}$$

Thus, within the SM, if the $t$-quark dominates the $b \to d$ penguin, one expects the CP asymmetry in $B_d^0(t) \to J/\Psi K_S$ to be equal to that in $B_s^0(t) \to \phi K_S$ [14].

If there is new physics, the CP asymmetry in $B_s^0(t) \to \phi K_S$ can be affected in two ways: there may be new contributions to $B_s^0$-$\overline{B_s^0}$ mixing (the $b \to s$ FCNC) and/or to the penguin decay $B_s^0 \to \phi K_S$ (the $b \to d$ FCNC). As discussed previously, new physics in $B_s^0$-$\overline{B_s^0}$ mixing can be discovered independently, for example by comparing the CP asymmetries in $B^\pm \to D K^\pm$ and $B_s^0(t) \to D_s^\pm K^\mp$. If, after taking this into account, there is still a discrepancy in the value of $\beta$ as extracted from the CP asymmetries in $B_d^0(t) \to J/\Psi K_S$ and $B_s^0(t) \to \phi K_S$, this will indicate the presence of new physics in the $b \to d$ FCNC.

Note also that one can perform a similar analysis with the related decay $B_d^0 \to K^0 \overline{K^0}$ [14]. (At the quark level, the decays are identical, save for the flavour of the spectator quark; at the meson level there is a difference since here there are two pseudoscalars in the final state, while the $B_s^0$ decay has a vector and a pseudoscalar.) Under the same assumptions as above, the CP asymmetry in $B_d^0(t) \to K^0 \overline{K^0}$ measures

$$\mathrm{Arg}\left[\frac{V_{tb}^* V_{td}}{(V_{tb}^* V_{td})^2 V_{tb} V_{td}^*}\right] = 0 . \tag{2}$$

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Thus, a non-zero CP asymmetry in this mode would indicate the presence of new physics in the $b \to d$ FCNC.

However, there is a problem with the above analysis. Theoretical estimates suggest that the $b \to d$ penguin is *not* dominated by an internal $t$-quark. On the contrary, the $u$- and $c$-quark contributions can be substantial, perhaps even as large as 20%–50% of the $t$-quark contribution [15]. If this is the case, then the CP asymmetry in $B_s^0(t) \to \phi K_S$ no longer cleanly probes the angle $\beta$. Instead, there is now "penguin pollution" from the $u$- and $c$-quark contributions to the $b \to d$ penguin, and the result depends on (unknown) hadronic quantities such as the strong phases and the relative sizes of the various penguin contributions. Thus, if one wants to detect new physics in the $b \to d$ FCNC, it is necessary to deal with this penguin pollution. As we show below, this can be done by combining information from both $B_s^0(t) \to \phi K_S$ and $B_d^0 \to K^0 \overline{K^0}$.

The $B_s$ and $B_d$ systems differ in that the width difference between the light and heavy $B_s$ eigenstates may be measurable, which is not the case for the $B_d$ system. In the presence of a non-negligible width difference, the expressions for the time-dependent decays of $B_s$ mesons are [16]

$$
\begin{aligned}
\Gamma(B_s^0(t) \to f) &= |A_f|^2 |g_+(t)|^2 + |\bar{A}_f|^2 |g_-(t)|^2 \\
&\quad + 2\left[ \mathrm{Re}(A_f^* \bar{A}_f) \mathrm{Re}(g_-(t) g_+^*(t)) - \mathrm{Im}(A_f^* \bar{A}_f) \mathrm{Im}(g_-(t) g_+^*(t)) \right] , \\
\Gamma(\overline{B_s^0}(t) \to f) &= |\bar{A}_f|^2 |g_+(t)|^2 + |A_f|^2 |g_-(t)|^2 \\
&\quad + 2\left[ \mathrm{Re}(A_f^* \bar{A}_f) \mathrm{Re}(g_-(t) g_+^*(t)) + \mathrm{Im}(A_f^* \bar{A}_f) \mathrm{Im}(g_-(t) g_+^*(t)) \right] , \quad (3)
\end{aligned}
$$

with

$$
\begin{aligned}
|g_+(t)|^2 + |g_-(t)|^2 &= \frac{1}{2}\left( e^{-\Gamma_L t} + e^{-\Gamma_H t} \right) , \\
|g_+(t)|^2 - |g_-(t)|^2 &= e^{-\Gamma t} \cos \Delta m t , \\
\mathrm{Re}[g_-(t) g_+^*(t)] &= \frac{1}{4}\left( e^{-\Gamma_L t} - e^{-\Gamma_H t} \right) , \\
\mathrm{Im}[g_-(t) g_+^*(t)] &= \frac{1}{2} e^{-\Gamma t} \sin \Delta m t . \quad (4)
\end{aligned}
$$

In the above, $\Gamma_L$ and $\Gamma_H$ are the widths of the light and heavy $B$-states, respectively, and $\Gamma \equiv (\Gamma_L + \Gamma_H)/2$. (Note also that we have assumed that the weak phase in $B_s^0$-$\overline{B_s^0}$ mixing is zero, which holds within the SM.)

Thus, the time-dependent measurements of $B_s$ decay rates allow one to obtain the following four functions of the decay amplitudes:

$$|A_f|^2 + |\bar{A}_f|^2 \quad , \quad |A_f|^2 - |\bar{A}_f|^2 \quad , \quad \text{Re}\left(A_f^*\bar{A}_f\right) \quad , \quad \text{Im}\left(A_f^*\bar{A}_f\right). \tag{5}$$

If the width difference in the $B_s$ system turns out to be small, then $\text{Re}[g_-(t)g_+^*(t)] \simeq 0$, which appears to imply that one cannot obtain the quantity $\text{Re}\left(A_f^*\bar{A}_f\right)$. However, this is not true. In fact, the above four functions are not independent, due to the equality

$$|A_f|^2|\bar{A}_f|^2 = [\text{Re}(A_f^*\bar{A}_f)]^2 + [\text{Im}(A_f^*\bar{A}_f)]^2 . \tag{6}$$

Thus, even if the width difference in the $B_s$ system is not measurable, we can still obtain each of the four quantities in Eq. (5), except that the *sign* of $\text{Re}\left(A_f^*\bar{A}_f\right)$ is undetermined. The lack of knowledge of this sign simply leads to additional possible solutions (discrete ambiguities), one for each sign of $\text{Re}\left(A_f^*\bar{A}_f\right)$. A measurable width difference allows the determination of this sign, thereby reducing discrete ambiguities. In what follows, we assume that the width difference is measurable, i.e. that the sign of $\text{Re}\left(A_f^*\bar{A}_f\right)$ is known. If this sign cannot be determined, the method is still valid, but extra solutions are possible.

Now, consider again the process $B_s^0 \to \phi K_S$. At the level of quark diagrams, there are several contributions to this decay: (i) the ordinary gluonic penguin, $\tilde{P}$, (ii) the Zweig-suppressed gluonic decay $\tilde{P}_1$, in which the gluon essentially hadronizes into the $\phi$, (iii) the electroweak penguin $\tilde{P}_{EW}$, and (iv) the colour-suppressed electroweak penguin $\tilde{P}_{EW}^C$. (The tildes on the amplitudes indicate a $B_s^0$ decay.) We therefore write the amplitude schematically as

$$A_s^\phi \equiv A\left(B_s \to \phi K_S\right) = \frac{1}{\sqrt{2}}\left(\tilde{P} + \tilde{P}_1 + \tilde{P}_{EW} + \tilde{P}_{EW}^C\right) . \tag{7}$$

(The factor $1/\sqrt{2}$ is included due to the presence of the $K_S$.) Of these, the gluonic penguins receive contributions from internal $u$, $c$ and $t$-quarks, while the electroweak penguins are $t$-quark dominated.

Any contribution to the $b \to d$ penguin can be written generically as

$$\begin{aligned} P = \sum_{q=u,c,t} V_{qb}^*V_{qd}P_q &= V_{cb}^*V_{cd}(P_c - P_u) + V_{tb}^*V_{td}(P_t - P_u) \\ &\equiv \mathcal{P}_{cu}e^{i\delta_c} + \mathcal{P}_{tu}e^{i\delta_t}e^{-i\beta} . \end{aligned} \tag{8}$$

In the first line we have used the unitarity of the CKM matrix to eliminate the $u$-quark contribution, and in the second we have explicitly separated out the weak and strong phases and absorbed the magnitudes $|V_{cb}^*V_{cd}|$ and $|V_{tb}^*V_{td}|$ into the definitions of $\mathcal{P}_{cu}$ and $\mathcal{P}_{tu}$, respectively.

Applying this to $B_s^0 \to \phi K_S$, Eq. (7) becomes

$$A_s^\phi = \frac{1}{\sqrt{2}} \left( \tilde{\mathcal{P}}_{cu} e^{i\tilde{\delta}_c} + \tilde{\mathcal{P}}_{tu} e^{i\tilde{\delta}_t} e^{-i\beta} \right), \tag{9}$$

where $\tilde{\mathcal{P}}_{cu}$ and $\tilde{\mathcal{P}}_{tu}$ are real and taken to be positive, and

$$
\begin{aligned}
\tilde{\mathcal{P}}_{cu} e^{i\tilde{\delta}_c} &= \tilde{P}_c - \tilde{P}_u + \tilde{P}_{1,c} - \tilde{P}_{1,u} , \\
\tilde{\mathcal{P}}_{tu} e^{i\tilde{\delta}_t} e^{-i\beta} &= \tilde{P}_t - \tilde{P}_u + \tilde{P}_{1,t} - \tilde{P}_{1,u} + \tilde{P}_{EW} + \tilde{P}_{EW}^C .
\end{aligned}
\tag{10}
$$

With this expression for the $B_s^0 \to \phi K_S$ amplitude, the measurements of the quantities in Eq. (5) give

$$
\begin{aligned}
\tilde{X} &\equiv \frac{1}{2} \left( \left| A_s^\phi \right|^2 + \left| \bar{A}_s^\phi \right|^2 \right) = \frac{1}{2} \left( \tilde{\mathcal{P}}_{cu}^2 + \tilde{\mathcal{P}}_{tu}^2 + 2\tilde{\mathcal{P}}_{cu}\tilde{\mathcal{P}}_{tu} \cos \tilde{\Delta} \cos \beta \right), \tag{11} \\
\tilde{Y} &\equiv \frac{1}{2} \left( \left| A_s^\phi \right|^2 - \left| \bar{A}_s^\phi \right|^2 \right) = \frac{1}{2} \left( -2\tilde{\mathcal{P}}_{cu}\tilde{\mathcal{P}}_{tu} \sin \tilde{\Delta} \sin \beta \right), \tag{12} \\
\tilde{Z}_R &\equiv Re \left( A_s^{\phi*} \bar{A}_s^\phi \right) = \frac{1}{2} \left( -\tilde{\mathcal{P}}_{cu}^2 - \tilde{\mathcal{P}}_{tu}^2 \cos 2\beta - 2\tilde{\mathcal{P}}_{cu}\tilde{\mathcal{P}}_{tu} \cos \tilde{\Delta} \cos \beta \right), \tag{13} \\
\tilde{Z}_I &\equiv Im \left( A_s^{\phi*} \bar{A}_s^\phi \right) = \frac{1}{2} \left( -\tilde{\mathcal{P}}_{tu}^2 \sin 2\beta - 2\tilde{\mathcal{P}}_{cu}\tilde{\mathcal{P}}_{tu} \cos \tilde{\Delta} \sin \beta \right), \tag{14}
\end{aligned}
$$

where $\tilde{\Delta} \equiv \tilde{\delta}_c - \tilde{\delta}_t$. (Note: in the above we have assumed that there is no new physics in $B_s^0$-$\overline{B_s^0}$ mixing. As mentioned previously, the presence of such new physics can be independently determined. We will discuss the case of new physics in $B_s^0$-$\overline{B_s^0}$ mixing further on.)

Examining the above equations, we note that there are four unknown parameters ($\tilde{\mathcal{P}}_{cu}$, $\tilde{\mathcal{P}}_{tu}$, $\tilde{\Delta}$ and $\beta$), but only three independent measurements. Thus, we cannot solve for the unknowns. In particular, we see that we cannot obtain $\beta$ in the presence of penguin pollution.

However, progress can be made if we also consider the decay $B_d^0 \to K^0\overline{K^0}$, which is similar to $B_s^0 \to \phi K_S$. (At the quark level, they differ only in the flavour of the spectator quark.) For this decay, contributions come only from the ordinary gluonic penguin $P$ and the colour-suppressed electroweak penguin $P_{EW}^C$ (amplitudes without tildes indicate $B_d^0$ decays):

$$A_d^{KK} = A \left( B_d^0 \to K^0\overline{K^0} \right) = P + P_{EW}^C . \tag{15}$$

Analogous to the $B^0_s \rightarrow \phi K_S$ amplitude, we can write

$$A^{KK}_d = \mathcal{P}_{cu} e^{i\delta_c} + \mathcal{P}_{tu} e^{i\delta_t} e^{-i\beta}, \quad (16)$$

where

$$
\begin{aligned}
\mathcal{P}_{cu} e^{i\delta_c} &= P_c - P_u , \\
\mathcal{P}_{tu} e^{i\delta_t} e^{-i\beta} &= P_t - P_u + P^C_{EW} .
\end{aligned}
\quad (17)
$$

In the $B^0_d$ system, the width difference is negligible, so that only three quantities can be obtained from time-dependent measurements:

$$X \equiv \frac{1}{2}\left(|A^{KK}_d|^2 + |\bar{A}^{KK}_d|^2\right) = \mathcal{P}^2_{cu} + \mathcal{P}^2_{tu} + 2\mathcal{P}_{cu}\mathcal{P}_{tu}\cos\Delta\cos\beta , \quad (18)$$

$$Y \equiv \frac{1}{2}\left(|A^{KK}_d|^2 - |\bar{A}^{KK}_d|^2\right) = -2\mathcal{P}_{cu}\mathcal{P}_{tu}\sin\Delta\sin\beta , \quad (19)$$

$$
\begin{aligned}
Z &\equiv Im\left(e^{-2i\tilde{\beta}} A^{KK*}_d \bar{A}^{KK}_d\right) \\
&= -\mathcal{P}^2_{cu}\sin 2\tilde{\beta} - \mathcal{P}^2_{tu}\sin(2\tilde{\beta} - 2\beta) - 2\mathcal{P}_{cu}\mathcal{P}_{tu}\cos\Delta\sin(2\tilde{\beta} - \beta) ,
\end{aligned}
\quad (20)
$$

where $\Delta \equiv \delta_c - \delta_t$. Since we are allowing for the possibility of new physics in the $b \rightarrow d$ FCNC, we have explicitly denoted the weak phase of $B^0_d$-$\overline{B^0_d}$ mixing as $\tilde{\beta}$, which may be different from the weak phase of the $b \rightarrow d$ penguin $\beta$.

It is reasonable to assume that the mixing phase $\tilde{\beta}$ will be measured independently via $B^0_d(t) \rightarrow J/\Psi K_S$. Even so, we are still left with three equations in four unknowns ($\mathcal{P}_{cu}$, $\mathcal{P}_{tu}$, $\Delta$ and $\beta$), so once again we cannot solve for $\beta$.

However, we can reduce the number of independent parameters in the $B^0_s(t) \rightarrow \phi K_S$ and $B^0_d(t) \rightarrow K^0\overline{K^0}$ measurements by making an assumption. Specifically, we assume that $r = \tilde{r}$, where $r \equiv \mathcal{P}_{cu}/\mathcal{P}_{tu}$ and $\tilde{r} \equiv \tilde{\mathcal{P}}_{cu}/\tilde{\mathcal{P}}_{tu}$. How good is this assumption? From Eqs. (10) and (17) we have

$$
\begin{aligned}
r &= \left|\frac{P_c - P_u}{P_t - P_u + P^C_{EW}}\right| \\
\tilde{r} &= \left|\frac{\tilde{P}_c - \tilde{P}_u + \tilde{P}_{1,c} - \tilde{P}_{1,u}}{\tilde{P}_t - \tilde{P}_u + \tilde{P}_{1,t} - \tilde{P}_{1,u} + \tilde{P}_{EW} + \tilde{P}^C_{EW}}\right| .
\end{aligned}
\quad (21)
$$

Now, at the quark level the only difference between the $P$ and the $\tilde{P}$ amplitudes is the flavour of the spectator quark. Since this flavour should not have a significant effect on the size of

the amplitude, for a given type of penguin contribution we can take $|P_i| \simeq |\tilde{P}_i|$. Furthermore, we can estimate the relative sizes of the various types of penguin contribution: $|\tilde{P}_{EW}/\tilde{P}_t| \simeq |\tilde{P}_{1,q}/\tilde{P}_q| \simeq |\tilde{P}^C_{EW}/\tilde{P}_{EW}| \simeq \bar{\lambda}$, where $\bar{\lambda} \sim 20\%$. Thus, we find that

$$r \simeq \tilde{r} = \left| \frac{P_c - P_u}{P_t - P_u} \right| \tag{22}$$

and

$$\frac{r - \tilde{r}}{r} = O(\bar{\lambda}) . \tag{23}$$

Taking $r = \tilde{r}$ is therefore a reasonable assumption.

With the assumption that $r = \tilde{r}$, the measurements take the form

$$\tilde{X} = \frac{1}{2}\tilde{\mathcal{P}}^2_{tu}[1 + 2r\cos\tilde{\Delta}\cos\beta + r^2] , \tag{24}$$

$$\tilde{Y} = \frac{1}{2}\tilde{\mathcal{P}}^2_{tu}[-2r\sin\tilde{\Delta}\sin\beta] , \tag{25}$$

$$\tilde{Z}_R = \frac{1}{2}\tilde{\mathcal{P}}^2_{tu}[-\cos 2\beta - 2r\cos\tilde{\Delta}\cos\beta - r^2] , \tag{26}$$

$$\tilde{Z}_I = \frac{1}{2}\tilde{\mathcal{P}}^2_{tu}[-\sin 2\beta - 2r\cos\tilde{\Delta}\sin\beta] , \tag{27}$$

$$X = \mathcal{P}^2_{tu}[1 + 2r\cos\Delta\cos\beta + r^2] , \tag{28}$$

$$Y = \mathcal{P}^2_{tu}[-2r\sin\Delta\sin\beta] , \tag{29}$$

$$Z = \mathcal{P}^2_{tu}[-\sin(2\tilde{\beta} - 2\beta) - 2r\cos\Delta\sin(2\tilde{\beta} - \beta) - r^2\sin 2\tilde{\beta}] . \tag{30}$$

Assuming that $\tilde{\beta}$ is measured in $B^0_d(t) \to J/\psi K_S$, we now have six independent equations in six unknowns.

We can solve for $\beta$ as follows. First, $\mathcal{P}_{tu}$ and $\tilde{\mathcal{P}}_{tu}$ are eliminated by dividing the equations as follows:

$$\tilde{M} \equiv -\frac{\tilde{Z}_R}{\tilde{X}} = \frac{\cos 2\beta + 2r\cos\tilde{\Delta}\cos\beta + r^2}{1 + 2r\cos\tilde{\Delta}\cos\beta + r^2} = -1 + \frac{2\sin^2\beta}{1 + 2r\cos\tilde{\Delta}\cos\beta + r^2} \tag{31}$$

$$\tilde{N} \equiv \frac{\tilde{Z}_I}{\tilde{X}} = \frac{\sin 2\beta + 2r\cos\tilde{\Delta}\sin\beta}{1 + 2r\cos\tilde{\Delta}\cos\beta + r^2} = -2\sin\beta\frac{\cos\beta + r\cos\tilde{\delta}}{1 + 2r\cos\tilde{\Delta}\cos\beta + r^2} \tag{32}$$

$$\tilde{O} \equiv \frac{\tilde{Y}}{\tilde{X}} = \frac{-2r\sin\tilde{\Delta}\sin\beta}{1 + 2r\cos\tilde{\Delta}\cos\beta + r^2} \tag{33}$$

$$M \equiv \frac{Z}{X} + \sin 2\tilde{\beta} = \sin 2\tilde{\beta}(\frac{2\sin^2\beta}{1 + 2r\cos\Delta\cos\beta + r^2}) + \cos 2\tilde{\beta}\frac{\sin 2\beta + 2r\cos\Delta\sin\beta}{1 + 2r\cos\Delta\cos\beta + r^2} \tag{34}$$

$$O \equiv \frac{Y}{X} = \frac{-2r\sin\Delta\sin\beta}{1 + 2r\cos\Delta\cos\beta + r^2} \tag{35}$$

We then define

$$R \equiv -\frac{\tilde{M}+1}{\tilde{N}} = \frac{\sin\beta}{\cos\beta + r\cos\tilde{\Delta}} \qquad (36)$$

Eliminating $\tilde{\delta}$ from Eqs. (36) and (31), we then find $r^2$ as a function of $\beta$:

$$r^2 = -1 + 2\cos^2\beta + \frac{2}{\tilde{M}+1}\sin^2\beta - \frac{2}{R}\sin\beta\cos\beta \qquad (37)$$

And from Eq. (34) we have

$$\cos\Delta = \frac{-2\sin 2\tilde{\beta}\sin^2\beta - 2\cos 2\tilde{\beta}\sin\beta\cos\beta + M + r^2 M}{2r(-M\cos\beta + \cos 2\tilde{\beta}\sin\beta)} \qquad (38)$$

Finally, by inserting the expressions for $\Delta$ and $r^2$ into Eq. (35) we obtain an equation for $\beta$ in terms of observables alone. Note that we have not used the expression for $\tilde{Y}$ [Eq. (25)] in the above derivation. As explained earlier, $\tilde{Y}$ is not independent of $\tilde{X}$, $\tilde{Z}_R$ and $\tilde{Z}_I$. However, it can be used as a check to eliminate some of the solutions, thereby reducing the discrete ambiguity.

We illustrate this solution numerically in Table 1. By choosing input values for the theoretical parameters, we can generate the "experimental data" of Eqs. (24)-(30). The above method can then be used to solve for the theoretical unknowns, and we can check that, despite the presence of multiple solutions, we can still find that $\beta \neq \tilde{\beta}$. For the amplitudes, we take $(\tilde{\mathcal{P}}_{tu})_{in} = 1.0$ and $(\mathcal{P}_{tu})_{in} = 1.2$. (We take these quantities to be unequal in order to account for two things: (i) the different spectator quarks and (ii) the different final state – two pseudoscalars in one case, and one vector and one pseudoscalar in the other case.) The assumed input values of $r$ and the weak and strong phases are shown in the Table. The angles are taken to lie in the region $0 < \beta, \Delta, \tilde{\Delta} < \pi$.

From the Table, we see that $\beta$ can be extracted with a fourfold ambiguity. However, none of these values is equal to the value of $\tilde{\beta}$. Thus, if these measurements were carried out, and these results found, we would have unequivocal evidence of new physics in the $b \to d$ FCNC. We would not know if it affected $B_d^0$-$\overline{B_d^0}$ mixing, the $b \to d$ penguin, or both, but we would know with certainty that new physics was present.

In describing this method, we have assumed that there is no new physics in $B_s^0$-$\overline{B_s^0}$ mixing. However, even if we include this, the above method does not change significantly. If a new-physics phase $\theta_s$ is present in $B_s^0$-$\overline{B_s^0}$ mixing, then it is not the quantities $\tilde{Z}_R$ and $\tilde{Z}_I$

| $\tilde{\beta}_{in}$ | $\beta_{in}$ | $r_{in}$ | $\tilde{\Delta}_{in}$ | $\Delta_{in}$ | $\beta$ | $r$ | $\tilde{\Delta}$ | $\Delta$ | $\tilde{P}_{tu}$ |
|---|---|---|---|---|---|---|---|---|---|
| 25 | 10 | 0.3 | 80 | 120 | 19.7 | 1.22 | 28.1 | 180.0 | 0.26 |
| | | | | | 10.0 | 0.30 | 80.0 | 120.0 | 1.00 |
| | | | | | 160.3 | 1.22 | 151.9 | 0.0 | 0.26 |
| | | | | | 170.0 | 0.30 | 100.0 | 60.0 | 1.00 |
| 25 | 40 | 0.3 | 80 | 120 | 40.0 | 0.30 | 80.0 | 120.0 | 1.00 |
| | | | | | 57.2 | 0.65 | 35.8 | 165.9 | 0.59 |
| | | | | | 140.0 | 0.30 | 100.0 | 60.0 | 1.00 |
| | | | | | 122.8 | 0.65 | 144.2 | 14.1 | 0.59 |
| 40 | 25 | 0.3 | 40 | 10 | 64.4 | 2.04 | 11.6 | 178.5 | 0.47 |
| | | | | | 25.0 | 0.30 | 40.0 | 10.0 | 1.00 |
| | | | | | 115.6 | 2,04 | 168.4 | 1.53 | 0.47 |
| | | | | | 155.0 | 0.30 | 168.3 | 170.0 | 1.0 |

Table 1: Output values of $\beta$, $r$, $\tilde{\Delta}$, $\Delta$ and $\tilde{P}_{tu}$ for given input values of $r$ and the weak and strong phases. We take $(\tilde{\mathcal{P}}_{tu})_{in} = 1.0$ and $(\mathcal{P}_{tu})_{in} = 1.2$. All phase angles are given in degrees.

[Eqs. (13) and (14)] which are measured, but rather $\tilde{Z}_R^{ex}$ and $\tilde{Z}_I^{ex}$:

$$\tilde{Z}_R^{ex} \equiv Re\left(e^{-i\theta_s} A_s^{\phi*}\bar{A}_s^{\phi}\right) = \cos\theta_s \tilde{Z}_I - \sin\theta_s \tilde{Z}_R , \tag{39}$$

$$\tilde{Z}_I^{ex} \equiv Im\left(e^{-i\theta_s} A_s^{\phi*}\bar{A}_s^{\phi}\right) = \cos\theta_s \tilde{Z}_R + \sin\theta_s \tilde{Z}_I . \tag{40}$$

However, $\tilde{Z}_R$ and $\tilde{Z}_I$ can be obtained straightforwardly:

$$\tilde{Z}_R = \cos\theta_s \tilde{Z}_I^{ex} + \sin\theta_s \tilde{Z}_R^{ex} \tag{41}$$

$$\tilde{Z}_I = -\sin\theta_s \tilde{Z}_I^{ex} + \sin\theta_s \tilde{Z}_R^{ex} \tag{42}$$

Thus, assuming that $\theta_s$ is known independently (e.g. via any of the methods we have described earlier), we can use these expressions for $\tilde{Z}_R$ and $\tilde{Z}_I$ and simply apply the above method. If $\theta_s$ is only known up to a discrete ambiguity, then this simply increases the number of possible solutions for $\beta$. However, in general we will still be able to determine that $\beta \neq \tilde{\beta}$.

Finally, we note that even if there is no new physics (i.e. $\beta = \tilde{\beta}$), this method still yields important information. If one probes $\beta$ in the conventional way via CP violation in $B_d^0(t) \rightarrow J/\Psi K_s$, one extracts the function $\sin 2\beta$. This gives the angle $\beta$ up to a fourfold discrete ambiguity: if $\beta_0$ is the true solution, $\beta_0 + \pi$, $\frac{\pi}{2} - \beta_0$ and $\frac{3\pi}{2} - \beta_0$ are also solutions for $\beta$. Our technique can be used to eliminate two of these solutions. In particular, due to the presence of the $\sin\beta$ and $\cos\beta$ factors in Eqs. (24)-(30), $\frac{\pi}{2} - \beta_0$ and $\frac{3\pi}{2} - \beta_0$ will not in

general be among the solutions to these equations. However, $\beta_0 + \pi$ will still be allowed if we simultaneously take $\Delta \to \Delta + \pi$ and $\tilde{\Delta} \to \tilde{\Delta} + \pi$. Thus, in the absence of new physics, the above method can be used to reduce the discrete ambiguity in $\beta$ from a fourfold one to a twofold one.

## 3 $B_s^0(t) \to J/\Psi K_S$ and $B_d^0(t) \to J/\Psi \pi^0$

It is not difficult to think of variations on the above method. As a second example, consider the decays $B_s^0 \to J/\Psi K_s$ and $B_d^0 \to J/\Psi \pi^0$. Both of these decays get contributions from colour-suppressed tree diagrams, Zweig-suppressed gluonic penguins and electroweak penguins. If the penguins are not too small compared to the tree diagram, it may be possible to extract $\beta$ and compare it with $\tilde{\beta}$.

The amplitudes for these decays can be written as

$$
A_s^\psi \equiv A(B_s \to J/\psi K_s) = \tilde{C} + \tilde{P}_1 + \tilde{P}_{EW} ~, \tag{43}
$$

$$
A_d^{\psi\pi} \equiv A\left(B_d \to J/\psi \pi^0\right) = C + P_1 + P_{EW} ~, \tag{44}
$$

where $\tilde{C}$ and $C$ are the colour-suppressed tree amplitudes in $B_s^0$ and $B_d^0$ decays, respectively. The combination of CKM matrix elements involved in these amplitudes is $V_{cb}^* V_{cd}$, which is real in the Wolfenstein parametrization.

As before, we use CKM unitarity to eliminate the $u$-quark piece of the penguin contributions, allowing us to write

$$
A_s^\psi \equiv \tilde{C} e^{i\tilde{\delta}_C} + \tilde{P}_1 e^{i\tilde{\delta}_P} e^{-i\beta} ~, \tag{45}
$$

$$
A_d^{\psi\pi} \equiv C e^{i\delta_C} + P_1 e^{i\delta_P} e^{-i\beta} ~, \tag{46}
$$

where

$$
\begin{aligned}
\tilde{C} e^{i\tilde{\delta}_C} &= \tilde{C} + \tilde{P}_{1,c} - \tilde{P}_{1,u} ~, \\
\tilde{P}_1 e^{i\tilde{\delta}_P} e^{-i\beta} &= \tilde{P}_{1,t} - \tilde{P}_{1,u} + \tilde{P}_{EW} \\
C e^{i\delta_C} &= C + P_{1,c} - P_{1,u} ~, \\
P_1 e^{i\delta_P} e^{-i\beta} &= P_{1,t} - P_{1,u} + P_{EW} ~.
\end{aligned} \tag{47}
$$

In terms of these quantities, the time-dependent measurements yield the following:

$$\tilde{X} \equiv \frac{1}{2}\left(\left|A_s^{\psi}\right|^2 + \left|\bar{A}_s^{\psi}\right|^2\right) = \tilde{\mathcal{C}}^2 + \tilde{\mathcal{P}}_1^2 + 2\tilde{\mathcal{C}}\tilde{\mathcal{P}}_1 \cos\tilde{\delta}\cos\beta \ , \tag{48}$$

$$\tilde{Y} \equiv \frac{1}{2}\left(\left|A_s^{\psi}\right|^2 - \left|\bar{A}_s^{\psi}\right|^2\right) = -2\tilde{\mathcal{C}}\tilde{\mathcal{P}}_1 \sin\tilde{\delta}\sin\beta \ , \tag{49}$$

$$\tilde{Z}_R \equiv Re\left(A_s^{\psi*}\bar{A}_s^{\psi}\right) = -\tilde{\mathcal{C}}^2 - \tilde{\mathcal{P}}_1^2 \cos 2\beta - 2\tilde{\mathcal{C}}\tilde{\mathcal{P}}_1 \cos\tilde{\delta}\cos\beta \ , \tag{50}$$

$$\tilde{Z}_I \equiv Im\left(A_s^{\psi*}\bar{A}_s^{\psi}\right) = -\tilde{\mathcal{P}}_1^2 \sin 2\beta - 2\tilde{\mathcal{C}}\tilde{\mathcal{P}}_1 \cos\tilde{\delta}\sin\beta \ , \tag{51}$$

$$X \equiv \frac{1}{2}\left(\left|A_d^{\psi\pi}\right|^2 + \left|\bar{A}_d^{\psi\pi}\right|^2\right) = \mathcal{C}^2 + \mathcal{P}_1^2 + 2\mathcal{C}\mathcal{P}_1 \cos\delta\cos\beta \ , \tag{52}$$

$$Y \equiv \frac{1}{2}\left(\left|A_d^{\psi\pi}\right|^2 - \left|\bar{A}_d^{\psi\pi}\right|^2\right) = -2\mathcal{C}\mathcal{P}_1 \sin\delta\sin\beta \tag{53}$$

$$Z \equiv Im\left(e^{-2i\tilde{\beta}}A_d^{\psi\pi*}\bar{A}_d^{\psi\pi}\right)$$
$$= -\mathcal{C}^2 \sin 2\tilde{\beta} - \mathcal{P}_1^2 \sin(2\tilde{\beta} - 2\beta) - 2\mathcal{C}\mathcal{P}_1 \cos\delta\sin(2\tilde{\beta} - \beta) \ , \tag{54}$$

where $\tilde{\delta} \equiv \tilde{\delta}_C - \tilde{\delta}_P$ and $\delta \equiv \delta_C - \delta_P$.

Once again, this gives us six independent equations in seven unknowns. However, we can reduce the number of parameters by assuming that $r = \tilde{r}$, where $r \equiv \mathcal{P}_1/\mathcal{C}$ and $\tilde{r} \equiv \tilde{\mathcal{P}}_1/\tilde{\mathcal{C}}$. Looking at Eq. (47), it is clear that this assumption is well-justified – in fact, within the spectator model, the equality is exact. In this case the observables become

$$\tilde{X} = \tilde{C}^2[1 + 2r\cos\tilde{\delta}\cos\beta + r^2] \tag{55}$$

$$\tilde{Y} = \tilde{C}^2[-2r\sin\tilde{\delta}\sin\beta] \tag{56}$$

$$\tilde{Z}_R = \tilde{C}^2[-1 - 2r\cos\tilde{\delta}\cos\beta - r^2\cos 2\beta] \tag{57}$$

$$\tilde{Z}_I = \tilde{C}^2[-2r\cos\tilde{\delta}\sin\beta - r^2\sin 2\beta] \tag{58}$$

$$X = C^2[1 + 2r\cos\delta\cos\beta + r^2] \tag{59}$$

$$Y = C^2[-2r\sin\delta\sin\beta] \tag{60}$$

$$Z = C^2[-\sin 2\tilde{\beta} - 2r\cos\delta\sin(2\tilde{\beta} - \beta) - r^2\sin(2\tilde{\beta} - 2\beta)] \tag{61}$$

The form of these equations is similar to that found for $B_s^0 \to \phi K_S$ and $B_d^0 \to K^0\overline{K^0}$ decays [Eqs. (24)-(30)]. As in that case, assuming that $\tilde{\beta}$ is measured in $B_d^0(t) \to J/\psi K_S$, we now have six independent equations in six unknowns. And as before, it is possible to solve these equations for the six parameters. We can thus obtain $\beta$, and test whether $\beta = \tilde{\beta}$ or not.

It must be admitted, however, that from a theoretical point of view this method is much less compelling than the one which uses the decays $B_s^0 \to \phi K_S$ and $B_d^0 \to K^0\overline{K^0}$. We have

noted that the assumption that $r = \tilde{r}$ is well justified since the only difference between the decays $B_s^0 \to J/\Psi K_s$ and $B_d^0 \to J/\Psi \pi^0$ is the flavour of the spectator quark. However, this is also problematic: if the flavour of the spectator quark is completely irrelevant, then we also have $\mathcal{C} = \tilde{\mathcal{C}}$ and $\delta = \tilde{\delta}$. But in this case the $B_d^0 \to J/\Psi \pi^0$ decay gives us no extra information: we have $X = \tilde{X}$, $Y = \tilde{Y}$, and $Z = \tilde{Z}_R \cos 2\tilde{\beta} - \tilde{Z}_I \sin 2\tilde{\beta}$. So we are back to the situation of having three equations in four unknowns, which obviously cannot be solved.

Thus, for this method to work, not only is it necessary for the penguin contributions to $B_s^0 \to J/\Psi K_s$ and $B_d^0 \to J/\Psi \pi^0$ to be sizeable, there must also be significant differences in the sizes of the contributing amplitudes due to the flavour of the spectator quark. While it is possible that these conditions are fulfilled, it is theoretically disfavoured. Thus, the method involving the decays $B_s^0 \to \phi K_s$ and $B_d^0 \to K^0 \overline{K^0}$ is probably more promising.

# 4 Conclusions

CP-violating asymmetries in $B$ decays will be measured in the near future. If Nature is kind, we will find evidence of physics beyond the SM. The most obvious signal of new physics will be if the unitarity triangle constructed from measurements of the CP angles disagrees with that constructed from independent measurements of the sides. The problem here is that there are significant theoretical uncertainties in the measurements of the sides. Thus, even if there is a discrepancy, it may not provide compelling evidence for new physics – it may simply be that the errors on the theoretical parameters have been underestimated. For this reason, it is important to find ways of directly probing for new physics in the $B$ system.

Although there are several methods which will allow us to directly test for the presence of new physics in the $b \to s$ FCNC (either in $B_s^0$-$\overline{B_s^0}$ mixing or in $b \to s$ penguins), finding new physics in the $b \to d$ FCNC ($B_d^0$-$\overline{B_d^0}$ mixing or $b \to d$ penguins) is considerably more difficult. To date, no methods have been suggested which directly probe new physics in the $b \to d$ FCNC.

In this paper we have discussed a method to search for new physics in the $b \to d$ FCNC. By making time-dependent measurements of the decays $B_s^0(t) \to \phi K_s$ and $B_d^0(t) \to K^0 \overline{K^0}$ it is possible to compare the weak phase in $B_d^0$-$\overline{B_d^0}$ mixing with that of the $t$-quark contribution to the $b \to d$ penguin. Since these phases are equal in the SM, any discrepancy would be a clear signal of new physics. The method is not entirely free of hadronic uncertainties: it does

require some theoretical input. Still, the necessary assumption — that a ratio of amplitudes in the two decays is equal — is reasonably well-justified theoretically. We estimate the error in this assumption to be $\lesssim 20\%$.

This method can be applied to other decay modes. For example, we have also examined the decays $B_s^0(t) \to J/\Psi K_s$ and $B_d^0(t) \to J/\Psi \pi^0$. Although in principle new physics in the $b \to d$ FCNC can be found using this set of decays, the necessary conditions are theoretically disfavoured. Thus the decays $B_s^0(t) \to \phi K_s$ and $B_d^0(t) \to K^0 \overline{K^0}$ are more promising.

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# References

[1] For reviews, see, for example, Y. Nir and H. R. Quinn in *B Decays*, edited by S. Stone (World Scientific, Singapore, 1994), p. 362; I. Dunietz, *ibid.*, p. 393; M. Gronau, *Proceedings of Neutrino 94, XVI International Conference on Neutrino Physics and Astrophysics*, Eilat, Israel, May 29 – June 3, 1994, eds. A. Dar, G. Eilam and M. Gronau, *Nucl. Phys. (Proc. Suppl.)* **B38** (1995) 136.

[2] C.O. Dib, D. London and Y. Nir, *Int. J. Mod. Phys.* **A6** (1991) 1253.

[3] Y. Grossman and M.P. Worah, *Phys. Lett.,* **B395** (1997) 241.

[4] M. Gronau and D. Wyler, *Phys. Lett.,* **B265** (1991) 172. See also M. Gronau and D. London, *Phys. Lett.,* **B253** (1991) 483; I. Dunietz, *Phys. Lett.,* **B270** (1991) 75. Improvements to this method have recently been discussed by D. Atwood, I. Dunietz and A. Soni, *Phys. Rev. Lett.,* **78** (1997) 3257.

[5] R. Aleksan, I. Dunietz and B. Kayser, *Z. Phys.,* **C54** (1992) 653.

[6] Y. Nir and D. Silverman, *Nucl. Phys.*, **B345** (1990) 301.

[7] For a review of new-physics effects in CP asymmetries in the $B$ system, see M. Gronau and D. London, *Phys. Rev.,* **D55** (1997) 2845, and references therein.

[8] See A. Ali and D. London, DESY 99-042, UdeM-GPP-TH-99-58, hep-ph/9903535, and references therein.

[9] D. London and A. Soni, *Phys. Lett.,* **B407** (1997) 61.

[10] A.E. Snyder and H.R. Quinn, *Phys. Rev.,* **D48** (1993) 2139.

[11] D. London, N. Sinha and R. Sinha, UdeM-GPP-TH-99-61, IMSc-99/05/18, hep-ph/9905404.

[12] N. Cabibbo, *Phys. Rev. Lett.,* **10** (1963) 531; M. Kobayashi and K. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.

[13] L. Wolfenstein, *Phys. Rev. Lett.,* **51** (1983) 1945.

[14] D. London and R.D. Peccei, *Phys. Lett.,* **B223** (1989) 257.

[15] A.J. Buras and R. Fleischer, *Phys. Lett.,* **B341** (1995) 379.

[16] I. Dunietz, *Phys. Rev.,* **D52** (1995) 3048.